10.3 Calculation for Cylindrical worm gear pair strength

Gear strength calculation formula for Cylindrical worm gear pair **JGMA 405-01 (1978)**

1. Applicable range (Common)

1.1 This standard is applied to Worm gear pair with following ranges and shaft angle 90° for power transfer used in general industrial machinery.

Axial module	: 1 ～ 25 mm
Reference diameter of Worm wheel	: Below 900 mm
Sliding velocity	: Below 30 m/s
Revolving velocity of Worm wheel	: Below 600 min^{-1}
Tooth profile	: Stipulated in JIS B1723 (Cylindrical worm gear pair)
Material	: Refer to Table 7

1.2. This standard is used for calculating Allowable load from given dimension of Cylindrical worm gear pair or is used for determining suitable dimensions of Cylindrical worm gear pair from given load.

2. Definition

Gear strength of Cylindrical worm gear pair is Allowable load for Surface durability.

3. Basic conversion formula and numerical value

3.1 Sliding velocity v_s (m/s)

$$v_s = \frac{d_1 n_1}{19100 \cos\gamma} \quad \cdots\cdots (1)$$

Hereby

d_1 : Reference pitch diameter of Worm gear (mm)
n_1 : Revolving velocity of Worm gear (min^{-1})
γ : Reference pitch cylindrical lead angle (°)

3.2 Torque, Tangential load and Efficiency

(1) When Worm gear is driver (speed reduction)

$$T_2 = \frac{F_t d_2}{2000} \text{(kgf・m)} \quad \cdots\cdots (2)$$

$$T_1 = \frac{T_2}{u \eta_R} = \frac{F_t d_2}{2000 u \eta_R} \text{(kgf・m)} \quad \cdots\cdots (3)$$

$$\eta_R = \frac{\tan\gamma\left(1 - \tan\gamma \dfrac{\mu}{\cos\alpha_n}\right)}{\tan\gamma + \dfrac{\mu}{\cos\alpha_n}} \quad \cdots\cdots (4)$$

Hereby

T_2 : Nominal torque (kgf・m) for Worm wheel
T_1 : Nominal torque (kgf・m) for Worm gear
F_t : Nominal Tangential load (kgf) on the Reference pitch circle for Worm wheel
d_2 : Reference pitch diameter (mm) for Worm wheel
u : Gear ratio
η_R : Transfer efficiency of Worm gear when Worm gear is driver (excludes bearing loss and mixer loss of lubricating oil)
μ : Friction factor {Refer to (3) of 3.2}
α_n : Normal reference pressure angle(°)

(2) When Worm wheel is driver (speed increment)

$$T_2 = \frac{F_t d_2}{2000} \text{(kgf・m)} \quad \cdots\cdots \text{Same as (2)}$$

$$T_1 = \frac{T_2 \eta_1}{u} = \frac{F_t d_2 \eta_1}{2000 u} \text{(kgf・m)} \quad \cdots\cdots (5)$$

$$\eta_1 = \frac{\tan\gamma - \dfrac{\mu}{\cos\alpha_n}}{\tan\gamma\left(1 + \tan\gamma \dfrac{\mu}{\cos\alpha_n}\right)} \quad \cdots\cdots (6)$$

Hereby

η_1 : Transfer efficiency of Worm gear pair when Worm wheel is driver (excludes bearing loss and mixer loss of lubricating oil).

(3) Numerical value of friction factor μ

Obtain Friction factor μ from Fig. 1 of sliding velocity when engaged with Worm gear with Case harden and ground or Worm wheel with phosphor bronze.

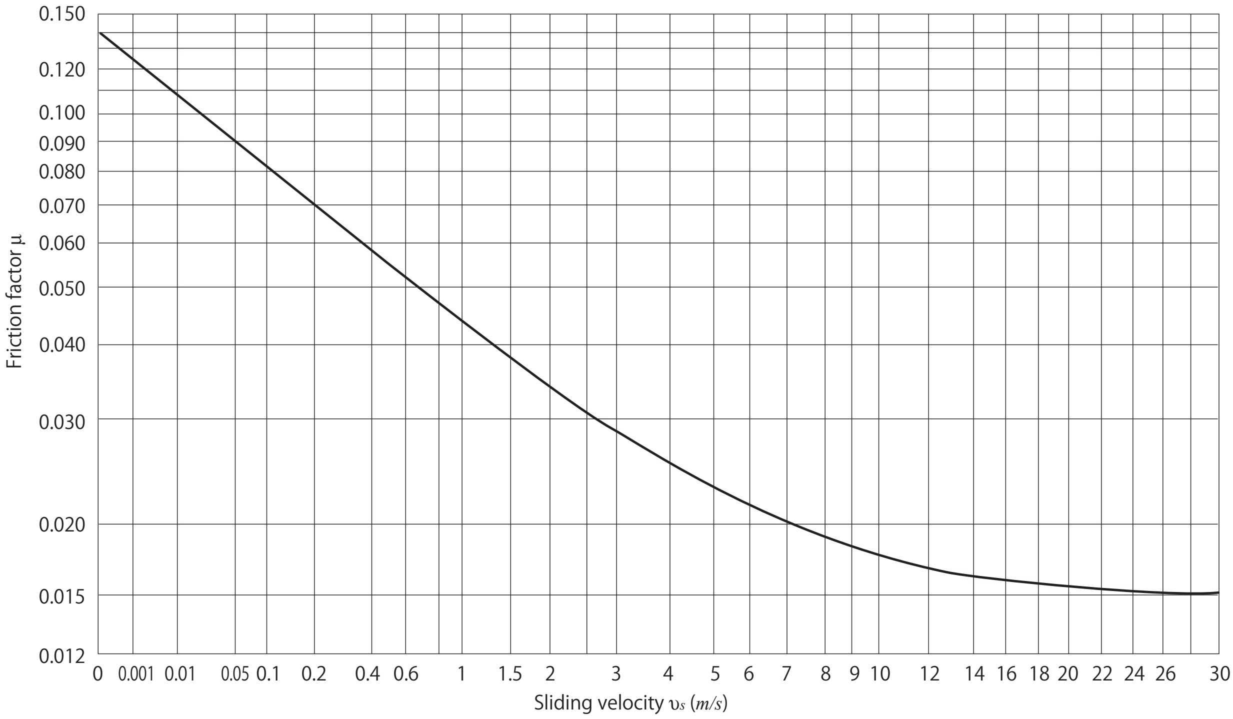
Remark 1. Friction factor for engagement with other materials.

Due to insufficient data, values of Friction factor are difficult to stipulate. Therefore Reference table 1 proposed by H.E Merritt is adopted for reference.

Reference table 1 Friction factor μ for different materials combination

Materials	Value of μ
Casting iron and phosphor bronze	1.15 times value of Fig. 1
Casting iron and Casting iron	1.33 times value of Fig. 1
Hardened steel and Aluminium	1.33 times value of Fig. 1
Steel and Steel	2.0 times value of Fig. 1

Fig. 1 Friction factor



4. Calculation formula of Allowable load for Surface durability

4.1 Basic load capacity calculation

Calculate Basic load capacity for Surface durability from given dimensions and material of Cylindrical worm gear pair using following calculation formula.

Allowable tangential load $F_{t\lim}$ (kg・f)

$$F_{t\lim} = 3.82 K_{\upsilon} K_{n} S_{c\lim} Z d_2^{0.8} m_x \frac{Z_L Z_M Z_R}{K_C} \quad \cdots\cdots(7)$$

Allowable Torque for Worm wheel $T_{2\lim}$ (kgf・m)

$$T_{2\lim} = 0.00191 K_{\upsilon} K_{n} S_{c\lim} Z d_2^{1.8} m_x \frac{Z_L Z_M Z_R}{K_C} \quad \cdots\cdots(8)$$

Hereby

d_2 : Reference pitch diameter (mm) for Worm wheel
m_x : Axial module (mm)
Z : Zone factor
K_{υ} : Sliding velocity factor
K_n : Revolving speed factor
Z_L : lubricating oil factor
Z_M : Lubrication factor
Z_R : Roughness factor
K_c : Tooth contact factor
$S_{c\lim}$: Allowable stress factor for Surface durability

4.2 Equivalent load calculation

Basic load capacity from formulas (7) and (8) is the limit of Tangential load and torque to withstand 26,000 hours of usage when in a non-impact environment. It is considered non impact if number of starts per hour is under 2 times and starting impact torque is below 200% of rated torque(1). However, if such condition is not met, calculate Equivalent load and compare with basic load capacity. In other words, when expected life is more or less than 26,000 hours with impact conditions applied. Starting toque is larger than above. Calculation method for Equivalent load is as follow.

Note(1) This is torque for Worm wheel when prime mover (or load) performs rated load operation.

Equivalent tangential load F_{te} (kgf)

$F_{te} = F_t K_h K_s$ ··························· (9)

Virtual torque of Worm wheel T_{2e} (kgf・m)

$T_{2e} = T_2 K_h K_s$ ··························· (10)

Hereby

F_t : Nominal tangential load on the Pitch circle of Worm wheel (kgf)

T_2 : Nominal torque of Worm wheel (kgf・m)

K_s : Starting factor (Refer to 5.9)

K_h : Time factor (Refer to 5.10)

4.3 Load definition

(1) When non-impact, expected life is 26,000 hours.
It should meet the following conditions.

$F_t \leqq F_{t\lim}$ ··························· (11)

$T_2 \leqq T_{2\lim}$ ··························· (12)

(2) Other than above cases,
it should meet the following conditions.

$F_{te} \leqq F_{t\lim}$ ··························· (13)

$T_{2e} \leqq T_{2\lim}$ ··························· (14)

Remark: For fluctuating load, use total torque T_{2c} to define load based on formulas (10) and (12) instead of T_2. Calculation method of T_{2c} is found at「Calculation of Fluctuating load」in Reference table 4 (page 153).

5. How to calculate each factor for Surface durability from calculation formula

Factors used for Surface durability calculation formulas mentioned above are stipulated below.

5.1 Facewidth of Worm wheel b_2 (mm)

Refer to Fig. 2 for Facewidth of Worm wheel.

5.2 Zone factor Z

Calculate Zone factor from (1) and (2) using Table 3.

(1) When , $b_2 < 2.3 m_x \sqrt{Q+1}$ use value in Table 3 multiplied by

$$\frac{b_2}{2 m_x \sqrt{Q+1}}$$ as value for Z.

(2) When $b_2 \geqq 2.3 m_x \sqrt{Q+1}$, use value in Table 3 multiplied by 1.15 as value for Z.

Hereby

Q : Diameter quotient $\left(Q = \frac{d_1}{m_x}\right)$

Z_w : Number of thread for Worm gear

5.3 Sliding velocity K_v

Obtain Sliding velocity factor based on Sliding velocity from Fig. 3.

5.4 Revolving velocity factor K_n

Obtain Revolving velocity factor based on Revolving speed of Worm wheel from Fig. 4.

5.5 Lubricating oil factor Z_L

As long as lubricating oil with proper viscosity containing extreme additives is used, $Z_L = 1.0$.

If bearing is used in Worm gear pair equipment or compelled to use lubricating oil with thin viscosity. Z_L is less than 1.0.

Remark: Viscosity

There are many recommended viscosity values from different sources for proper lubricating oil. However, there is no consensus. Recommended mean values are collected from sources and shown in Reference table 2.

Fig. 2 Facewidth of Worm wheel

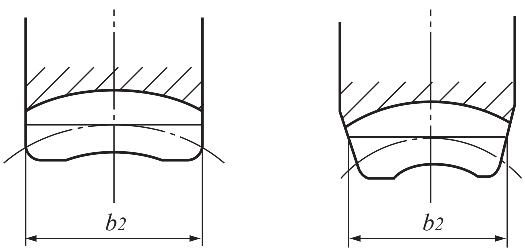


Table 3. Base value of Zone factor

Z_w \ Q	7	7.5	8	8.5	9	9.5	10	11	12	13	14
1	1.052	1.065	1.084	1.107	1.128	1.137	1.143	1.160	1.202	1.260	1.318
2	1.055	1.099	1.144	1.183	1.114	1.223	1.231	1.250	1.280	1.320	1.360
3	0.989	1.109	1.209	1.266	1.305	1.333	1.350	1.365	1.393	1.422	1.442
4	0.981	1.098	1.204	1.301	1.380	1.428	1.460	1.490	1.515	1.545	1.570

Reference table 2 Recommended dynamic viscosity

Unit cSt/37.8°C

Operating oil temperature		Sliding velocity m/s		
Max. oil temperature	Starting oil temperature	Below 2.5	Above 2.5 to below 5	Above 5
0 C° to below 10°C	−10°C to below 0C°	110 - 130	110 - 130	110 - 130
	Above 0°C	110 - 150	110 - 150	110 - 150
10 C° to below 30°C	Above 0°C	200 - 245	150 - 200	150 - 200
30 C° to below 55°C	〃	350 - 510	245 - 350	200 - 245
55 C° to below 80°C	〃	510 - 780	350 - 510	245 - 350
80 C° to below 100°C	〃	900 - 1100	510 - 780	350 - 510

Fig. 3 Sliding velocity factor

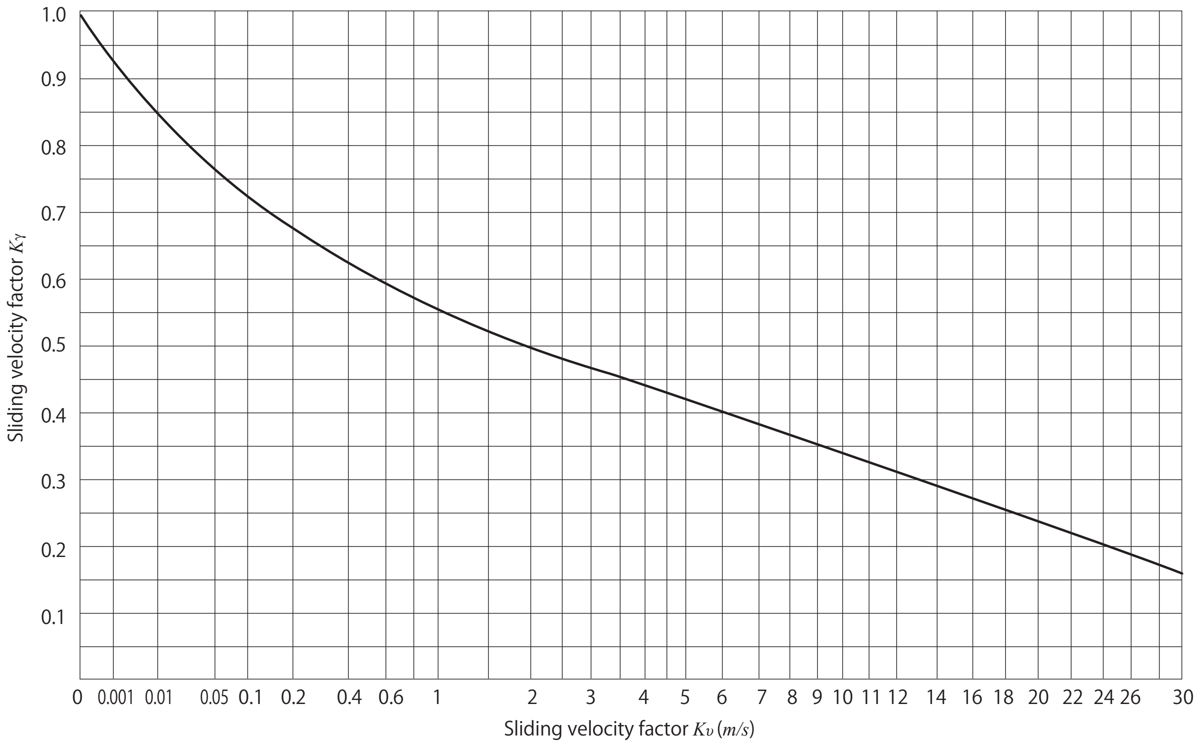
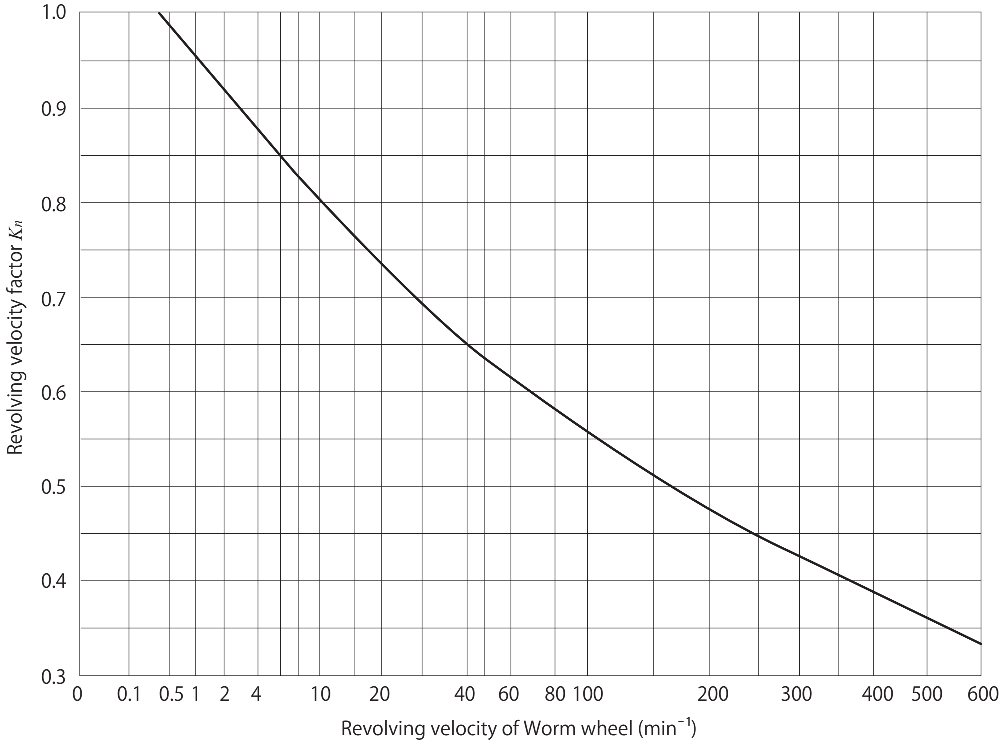


Fig. 4 Revolving velocity factor



5.6 Lubrication factor Z_M

Obtain Lubrication factor from Table 4.

Table 4. Lubrication factor Z_M

Sliding velocity m/s	Below 10	Above 10, below 14	Above 14
Oil bath lubrication	1.0	0.85	-
Forced lubrication	1.0	1.0	1.0

5.7 Roughness factor Z_R

Roughness factor is determined with consideration based on influence on Pitting and Wearing to flank of Worm gear and Worm wheel. Due to insufficient data, $Z_R = 1.0$ is adopted at the moment·························· (15)

However, Surface roughness is to be below 3S for Worm gear and below 12S for Worm wheel.

If Surface roughness is rougher than above, Roughness factor Z R should be lower than 1.0.

5.8 Tooth bearing K_c

Quality of Tooth bearing has large influence on load capacity. Due to insufficient data at the moment, Tooth bearing for classification equivalent to A in JIS B 1741 (tooth bearing) will be $K_c = 1.0$······················ (16)

Value of K_c for classification B and C is larger than 1.0. Reference table 3: Shows JIS Tooth bearing ratio and approximate values of K_c.

5.9 Starting factor K_s

Starting factor is stipulated below

(1) Obtain value from Table 5 if the starting torque is below 200% of rated torque.

(2) If Starting torque exceeds 200% of rated torque, value of $K_s = 1.0$. With starting torque to be as maximum, then calculate fluctuating load (refer to Table 4) to calculate total load.

5.10 Time factor K_h

Obtain Time factor from Table 6 using expected lifespan and extent of impact. Use interpolation when expected lifespan is between the values in the below Table.

Reference table 3 Classification of Tooth bearing and approximate value of K_c

Classification	Ratio of tooth bearing		K_c
	Tooth trace direction	Direction of tooth depth	
A	Above 50% of length of effective trace direction	Above 40% of effective tooth depth	1.0
B	Above 35% of length of effective trace direction	Above 30% of effective tooth depth	1.3 - 1.4
C	Above 20% of length of effective trace direction	Above 20% of effective tooth depth	1.5 - 1.7

Remark: Conditions for tooth bearing from JIS B1741

Table 5. Starting factor K_s

Number of start times per hour	Below 2 times	2 - 4 times	5 - 9 times	Above 10 times
K_s	1.0	1.07	1.13	1.18

Table 6. Time factor K_h

Impact from prime mover side	Expected lifespan	K_h		
		Impact from load		
		Uniform load	Medium impact	Heavy impact
Uniform load (Motor, Turbine, Hydraulic motor and others)	1,500 hours	0.80	0.90	1.0
	5,000 hours	0.90	1.0	1.25
	26,000 hours (1)	1.0	1.25	1.50
	60,000 hours	1.25	1.50	1.75
Light impact (Multiple cylinder engine)	1,500 hours	0.90	1.0	1.25
	5,000 hours	1.0	1.25	1.50
	26,000 hours (1)	1.25	1.50	1.75
	60,000 hours	1.50	1.75	2.0
Medium impact (Cylinder engine)	1,500 hours	1.0	1.25	1.50
	5,000 hours	1.25	1.50	1.75
	26,000 hours (1)	1.50	1.70	2.0
	60,000 hours	1.75	2.0	2.25

Note (1) Operating 10 hours a day for 260 days per a year is equivalent to 10 years and above.

5.11 Allowable stress factor S_{clim}

Table 7 shows Allowable stress factor and limits of sand burning sliding speed for Surface durability.

Table 7. Allowable stress factor S_{clim} for Surface durability

Material of Worm wheel	Material of Worm gear	Sclim	Limits of sand burning sliding velocity (1) m/s
Phosphor bronze centrifugal casting	Alloyed steel with Case hardening	1.55	30
	Alloyed steel HB400	1.34	20
	Alloyed steel HB250	1.12	10
Phosphor bronze chill casting	Alloyed steel with Case hardening	1.27	30
	Alloyed steel HB400	1.05	20
	Alloyed steel HB250	0.88	10
Phosphor bronze sand casting or Forging	Alloyed steel with Case hardening	1.05	30
	Alloyed steel HB400	0.84	20
	Alloyed steel HB250	0.70	10
Aluminum bronze	Alloyed steel with Case hardening	0.84	20
	Alloyed steel HB400	0.67	15
	Alloyed steel HB250	0.56	10
Bronze	Alloyed steel HB400	0.49	8
	Alloyed steel HB250	0.42	5
Graphite flake high strength casting	Same material as Worm wheel but with higher hardness.	0.70	5
Gray iron casting (Pearlite quality)	Phosphor bronze casting and Forging	0.63	2.5
	Same material as Worm wheel but with higher hardness.	0.42	2.5

Note (1): Values of S_{clim} in the table 7 is maximum sliding velocity applicable. Even if used below a calculated load, there is risk of sand burning if the sliding velocity exceeds this limit.

Remark 4 Calculation for Fluctuating load

(1)For combination of uniform torque with different revolving speeds,

When maximum nominal action T_{21}(1) operates Worm wheel at U_1 seconds per 1 cycle, smaller nominal torque T_{22}, $T_{23}\cdots$ at U_2, $U_3\cdots$ seconds and mean revolving speed is n_{21}, n_{22}, n_{23}, $\cdots$. calculate Equivalent time per 1 cycle based on T_{21} and n_{21} using below formula.

$$U_e = U_1 + U_2 \frac{n_{22}}{n_{21}} \left(\frac{T_{22}}{T_{21}} \right)^3 + U_3 \frac{n_{23}}{n_{21}} \left(\frac{T_{23}}{T_{21}} \right)^3 + \cdots\cdots \text{(R1)}$$

Hereby

U_e : Equivalent time (per 1 cycle) (s) based on T_{21} and n_{21}.

$n_{21} n_{22}\ n_{23}$ ···: mean revolving velocity of Worm wheel (min^{-1})

T_{21}, T_{22}, T_{23} ···: Nominal torque of Worm wheel (kgf・m)

Therefore Total equivalent time within 26,000 hours is as follow,

$$U_{ec} = \frac{U_e}{3600} \times (\text{Total number of cycle within 26,000 hours}) \cdots\cdots \text{(R2)}$$

Hereby, Total equivalent time per 26,000 hours based on U_{ec}・T_{21} and n_{21}.

Calculate Total torque from U_{ec} and Reference table 4 using the following formula.

$$T_{2c} = T_{21} K_h' \cdots\cdots \text{(R3)}$$

Hereby,

T_{2c} : Total sum of torques, T_{21}, T_{22}, $T_{23}\cdots$ (kgf・m)

K_h' : Factor taken from Reference table 4. If U_{ec} is median value, use interpolation.

Reference table 4 K_h'

U_{ee}	K_h	U_{ee}	K_h'
500 hours	0.77	5,000 hours	0.90
1,000 hours	0.79	10,000 hours	0.92
2,000 hours	0.81	25,000 hours	1.0
3,000 hours	0.84	26,000 hours	1.0

Note (1) : This table does not include torque peak with instantaneous change. Please use calculation formula from (2) for such types of torque peak.

Remark: When 1 cycle of the fluctuating load exactly matches one revolution of a Worm wheel, the largest torque always fall on only 1 specific tooth of the Worm wheel. Therefore calculation formula for fluctuating load is not applied. Calculated maximum torque is applied continuously to the whole expected lifespan.

Determine dimensions of Worm gear based on calculated Total torque T_{2c} from formula (R3) from (a) and (b).

(a) Non impact, expected lifespan is 26,000 hours. It is considered non impact if number of starts per hour is under 2 times and starting impact torque is below 200% of rated torque.

Detemine dimensions for worm gear pair in accordance with following relation.

$$T_{2c} \leqq T_{2\lim} \quad \text{(R4)}$$

Hereby

$T_{2\lim}$: Allowable torque for Worm wheel (kgf・m) to match with revolution velocity n_{21} for Worm wheel.

(b) When life is about 26,000 hours, impact conditions and number of start is above 2 times per hour. Design dimensions for Worm gear pair to form following relation.

$$T_{2c} K_h K_s \leqq T_{2\lim} \quad \text{(R5)}$$

Hereby $T_{2\lim}$: Allowable torque for Worm wheel (kgf•m) to match with revolution velocity n_{21} for Worm wheel.

(2) For combination of Peak torque and Flat torque when starting {Refer to 5.9 number(2)}.

Value of peak T_{21} during start reaches steady speed of operation after acceleration time of U_a seconds. If constant driving and torque are designated as T_{22}, Equivalent action time U_{1e} (s) is using following calculation.

$$U_{1e} = \frac{U_a}{4}\left(1+\frac{T_{22}}{T_{21}}\right)\left\{1+\left(\frac{T_{22}}{T_{21}}\right)^2\right\} \quad \text{(R6)}$$

Calculation of U_{1e}'(Torque peak equivalent action time per hour) with N times of start per hour is

$$U_{1e}' = NU_{1e} \quad \text{(R7)}$$

Actual time is NU_{1e}.

When such peak torque acts NU_a seconds per hour, steady torque T_{22} and Uniform torque T_{23},T_{24}···acts for U_2, U_3, U_4 seconds. When each mean revolution velocity is n_{21}, n_{22}, n_{23}, n_{24}, calculation of Equivalent time U_e(s) per hour is by following formula,

$$U_e = U_{1e}' + U_2\frac{n_{22}}{n_{21}}\left(\frac{T_{22}}{T_{21}}\right)^3 + U_3\frac{n_{23}}{n_{21}}\left(\frac{T_{23}}{T_{21}}\right)^3 + \quad \text{(R8)}$$

However, standard revolving speed n_{21} is the average value of peak torque between starting and end. Therefore, from standstill to reach n_{21} is calculated by $n_{21} = n'_{21} / 2$. T_{21} is standard torque. (Refer to Reference Fig. 1)

Total Virtual time in 26,000 hours is as follows.

$$U_{ec} = \frac{U_e}{3{,}600} \times 26000 \quad \text{(R9)}$$

Hereby

U_{ec} : Total equivalent time (h) per 26,000 hours based on T_{21} and n_{21}.

This U_{ec} is equivalent to U_{ec} of formula (R2) of previous item (1). Dimensions of Worm gear pair can be determined from formula (R3), (R4) or (R5) of (1) but K_s to be 1.0.

Reference Fig. 1 Conditions of Peak and Uniform torque