1.9 Contact ratio and Specific sliding

Contact ratio

(1) Theory of Contact ratio

Actual engaging teeth at working area are lesser than number of teeth manufactured on circumference.

Contact ratio describes working condition and is an element that influences gear oscillation, noise, strength, rotation and others.

It is generally believed that large Contact ratio is better. Below is the explanation using engagement between Spur gears as example.

Refer to the Fig. 27 for Involute cylindrical gear describes the engagement on the tangential line $\overline{I_1 I_2}$ of Base circle for both gears. This line is commonly called **Contact line** or **Line of action**.

Actual engagement on this Contact line is from range A_1 to A_2 of both Tip circles.

On the assumption that pinion is the driving gear. Firstly start contact between Dedendum of Pinion and tooth tip of gear at A_1 to engage.

As the gear rotates, point of contact passes through P-point (Pitch point), engaging with Dedendum of gear and Tooth tip of Pinion. After a short time, gears disengage at point A_2.

To perform gear rotation continuously, it is necessary for the next engaging pair of teeth to be engaged perfectly before disengaging the current pair.

In Fig. 27, $\overline{A_1 A_2} = g_a$ is called **Length of path of contact**. Distance from point A_1 to P is called Length of approach path g_α, distance point P to A_2 is called **Length of recess path** g_β.

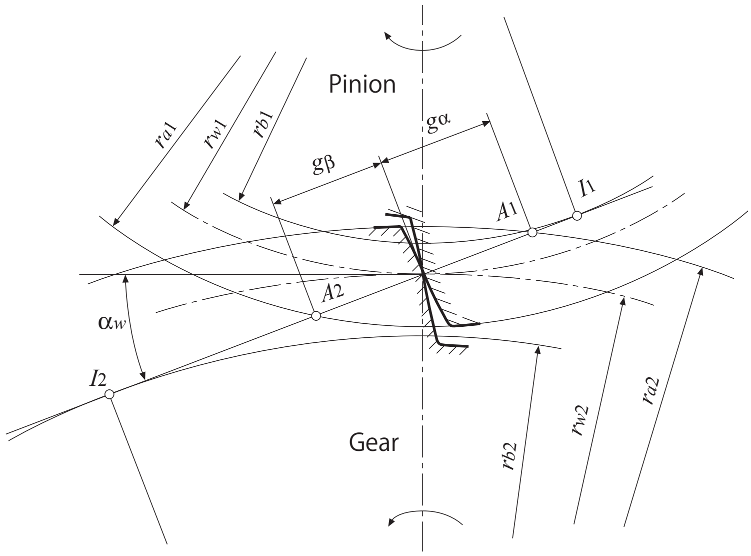

Fig. 27 Length of path of contact

Formula for Length of path of contact g is as follows.

$$g_\alpha = \overline{A_1P} = \overline{A_1I_2} - \overline{PI_2} = \sqrt{\gamma_{a2}{}^2 - \gamma_{b2}{}^2} - \gamma_{w2} \bullet \sin\alpha_w$$

$$g_\beta = \overline{A_2P} = \overline{A_2I_1} - \overline{PI_1} = \sqrt{\gamma_{a1}{}^2 - \gamma_{b1}{}^2} - \gamma_{w1} \bullet \sin\alpha_w$$

$$a_x = \gamma_{w1} + \gamma_{w2} \quad \text{Therefore}$$

$$g_a = g_\alpha + g_\beta = \sqrt{\gamma_{a2}{}^2 - \gamma_{b2}{}^2} + \sqrt{\gamma_{a1}{}^2 - \gamma_{b1}{}^2} - \alpha_x \bullet \sin\alpha_w$$

Hereby

γ_a :Tip radius
(The subscripts 1 and 2 indicate Pinion and Gear, respectively.)
γ_b : Base radius
α_w : Working pressure angle
α_x : Centre distance (Profile shifted gear)

Spacewidth on contact line is Base pitch ρ_b. **Contact ratio** is Length of path of contact divided by Base pitch. To maintain continuous rotation, Length of path of contact should be larger than Base pitch.
Therefore, formula of Contact ratio ε is as follows,

$$\varepsilon = \frac{\text{Length of path of contact}}{\text{Base Pitch}} = \frac{g_a}{\rho_b} \quad (\rho_b = \pi m \cos\alpha_0)$$

Contact ratio ε must be above 1.0

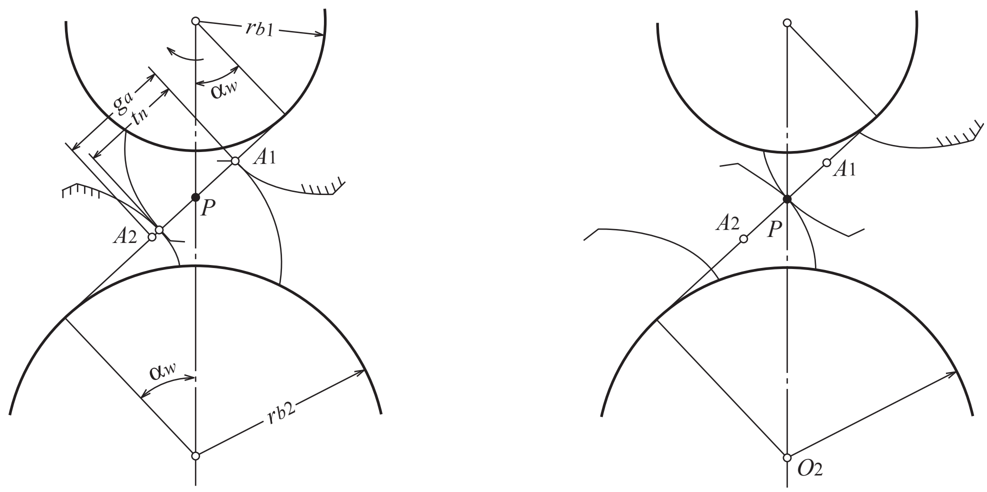

Fig. 28. Two teeth - contact and One tooth - contact.

For example, assume Contact ratio 1.487 for Spur gear pair engagement.
Look carefully at Fig. 28. In the beginning of engagement, engagement with two pairs of teeth. As two pairs rotate toward Pitch point, one pair of tooth is engaged.
When one pair of teeth continues rotating forward, two pairs of teeth engages. The cycle repeats.
Therefore, meaning of Contact ratio 1.487 is when two pairs of teeth will be engaged at 48.7% of Length on the path of contact with in the beginning and at the end. One pair of teeth will be engaged at the remaining 51.3%.
For gear with pressure angle 20°, repeating the same rotation when full load to one tooth and shared load to two teeth of gear.
Cause of oscillation and noise is due to the amount of deflection, which is different when engaging with one tooth or two teeth.
The value of Contact ratio depends on Pitch diameter, Pressure angle, Number of teeth, Rack shift coefficient and Tip diameter. Therefore refer to below.
1) Increase in Pressure angle will decrease Contact ratio.
2) Increase in sum (x_1+x_2) of Rack shift coefficient will decrease Contact ratio.
3) Full depth tooth gear with same Pressure angle and module will result in increase Contact ratio when Number of teeth is increased. On the other hand, when Number of teeth decreases and undercut occurs, Contact ratio will decrease extremely. Smaller Pressure angle will result in Contact ratio with a tendency to decrease.
4) When designing Full depth gear tooth (height of tooth is taller than full depth tooth), special tool is needed for the increased Tip diameter.

(2) Contact ratio of Spur gear

Refer to Table 15 for calculation formula for Contact ratio of Spur gear is as follows.

Assume the gear as a Rack, formula is $g_a=(h_{a2}-x_1m)/\sin\alpha_w$

Hereby

h_{a2} : Addendum of rack

x_1 : Rack shift coefficient of Spur gear

(3) Contact ratio for Helical gear

Contact ratio for Helical gear on the Transverse plane has the same calculation formula as Spur gear. Due to Helix tooth, value of Facewidth b divided by Normal pitch is added to Transverse contact ratio (This value is commonly called Overlap ratio).

Therefore,

The Transverse contact ratio ε_α + The Overlap ratio ε_β = The Total contact ratio ε_γ. Refer to Table 16, calculation formula of Contact ratio for Helical gear is as follows.

Table 15. Examples of Contact ratio for Spur gear

Common gear data: Module m=2.0, Cutter pressure angle α_0=20°

Gear 1	Gear 2		Contact ratio ε	Example
Spur gear $z_1=12$ $x_1=0.5$	Spur gear	$z_2=40$ $x_2=0$	$\varepsilon=\dfrac{\sqrt{\gamma_{a1}^2-\gamma_{b1}^2}+\sqrt{\gamma_{a2}^2-\gamma_{b2}^2}-\alpha_x\sin\alpha_w}{\pi m\cos\alpha_0}$	ε=1.399
	Rack	$x_2=0$	$\varepsilon=\dfrac{\sqrt{\gamma_{a1}^2-\gamma_{b1}^2}+\dfrac{h_{a2}-x_1m}{\sin\alpha_0}-\gamma_1\sin\alpha_0}{\pi m\cos\alpha_0}$	ε=1.475
	Internal gear	$z_2=100$ $x_2=0$	$\varepsilon=\dfrac{\sqrt{\gamma_{a1}^2-\gamma_{b1}^2}+\sqrt{\gamma_{a2}^2-\gamma_{b2}^2}+\alpha_x\sin\alpha_w}{\pi m\cos\alpha_0}$	ε=1.515

Table 16. Contact ratio of Helical gear

Common gear data: Normal module mn=2.0, Helix angle β=15°, Cutter pressure angle α_0=20°, Facewidth b=20.0.

Gear 1	Gear 2	Contact ratio ε	Example
$z=20$ $x_{n1}=0$	$z=40$ $x_{n2}=0$	Transverse contact ratio $\varepsilon_\alpha=\dfrac{\sqrt{\gamma_{a1}^2-\gamma_{b1}^2}+\sqrt{\gamma_{a2}^2-\gamma_{b2}^2}-\alpha_x\sin\alpha_{wt}}{\pi m_t\cos\alpha_t}$ Overlap ratio $\varepsilon_\beta=\dfrac{b\bullet\sin\beta}{\pi m_n}$ Total contact ratio $\varepsilon_\gamma=\varepsilon_\alpha+\varepsilon_\beta$	ε_α=1.561 ε_β=0.824 ε_γ=2.385

(4) Contact ratio for Bevel gear

Straight bevel gear uses the same calculation as Spur gear. To obtain Contact ratio, it assumes the formula of (1) Virtual spur gear upon the Back cone.
Due to Helix from tooth of Spiral bevel gear, overlap ratio is added to obtain the Transverse contact ratio from (1) Virtual spur gear for calculation.
Refer to Table 17 for calculation formula for Contact ratio of Bevel gear is as follows.

Table 17. Contact ratio for Bevel gear

Common gear data: Module m=2, Shaft angle Σ=90,
Face width b=13 (Spiral tooth) Pitch diameter d_1=36, d_2=72 Pitch angle δ_1=26° 33′ 54″, δ_2=63° 26′ 06″

Gear 1	Gear 2	Contact ratio ε	Example
z=18	z=36	Back cone distance $$R_v = \frac{d}{2 \cdot \cos\delta}$$	R_{v1}=20.125 R_{v2}=80.499
		Base radius of (1) Virtual spur gear (Straight tooth) $R_{vb} = R_v \cdot \cos\alpha_0$ (Spiral tooth) $R_{vb} = R_v \cdot \cos\alpha_t$	R_{vb1}=18.911 R_{vb2}=75.644 R_{vb1}=18.391 R_{vb2}=73.564
		Tip radius of (1) Virtual spur gear $R_{va} = R_v + h_a$	(Straight tooth) R_{va}=22.815 R_{va2}=81.809 (まがり歯) R_{va1}=22.410 R_{va2}=81.614
		Contact ratio (Straight tooth) $$\varepsilon = \frac{\sqrt{R_{va1}^2 - R_{vb1}^2} + \sqrt{R_{va2}^2 - R_{vb2}} - (R_{v1} + R_{v2})\sin\alpha_0}{\pi m \cos\alpha_0}$$	ε=1.610
		Transverse contact ratio (Spiral tooth) $$\varepsilon_\alpha = \frac{\sqrt{R_{va1}^2 - R_{vb1}^2} + \sqrt{R_{va2}^2 - R_{vb2}^2} - (R_{v1} + R_{v2})\sin\alpha_t}{\pi m \cos\alpha_t}$$ Overlap ratio $$\varepsilon_\beta = \frac{b\tan\beta_m}{\pi m} \cdot \frac{R_e}{R_e - 0.5b}$$ Total contact ratio $$\varepsilon_\gamma = \varepsilon_\alpha + \varepsilon_\beta$$	ε_α=1.270 ε_β=1.728 ε_γ=2.998

Note that (1) Adopted the old standard term.

Theory for Specific sliding (for reference)

Specific sliding is shown as condition of sliding where engaged flanks slides to transfer the rotation except area of pitch point.

Refer to Fig. 29, when one pair of Tooth profile is in contact at C point, after minute moment, it will contact points of C_1 and C_2 respectively. Where C-C_1=ds_1 and C-C_2=ds_2, calculation formula for Specific sliding δ is as follows.

$$\delta_1 = \frac{ds_1 - ds_2}{ds_1} \qquad \delta_2 = \frac{ds_2 - ds_1}{ds_2}$$

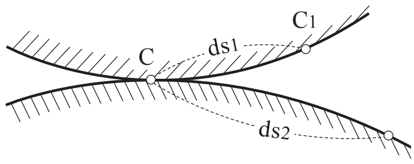

Fig. 29 Sliding

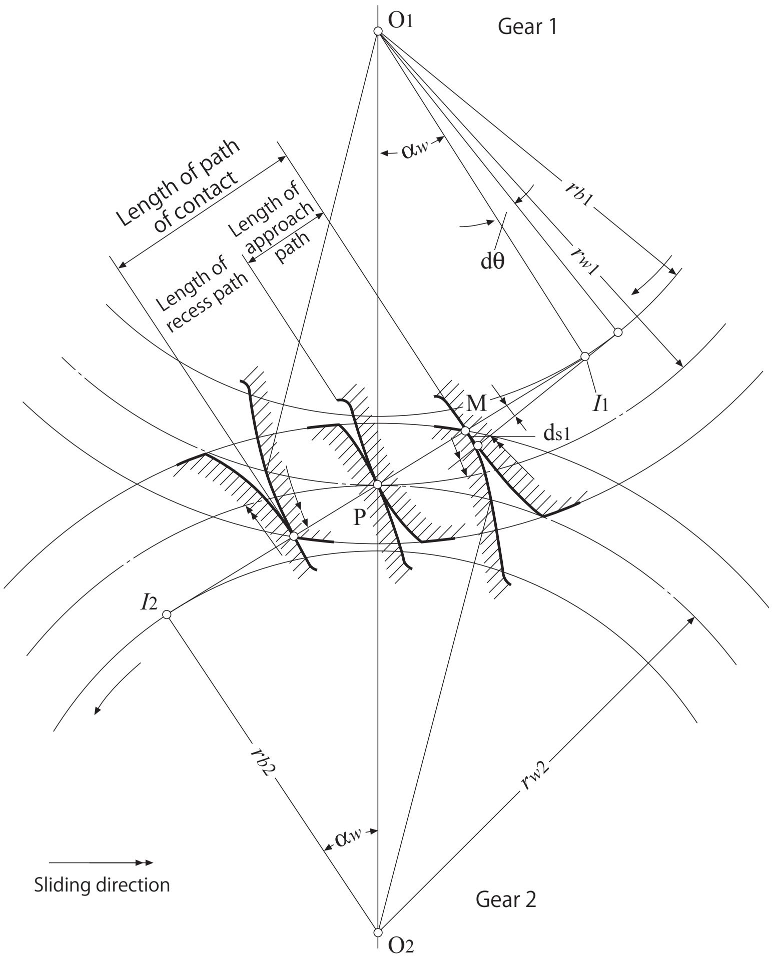

Fig. 30 Sliding direction of Flank for Involute tooth profile

Refer to Fig. 30, when Involute gear 1 makes $d\theta$ revolution as gear 2 makes $\gamma_1.d\theta/\gamma_2$ revolution.
When contact point upon Tooth profile has been shifted, length of ds_2 and ds_1 is by following formula,

$$ds_1 = (\overline{I_1M})d\theta \qquad ds_1 = (\overline{I_2M})\frac{\gamma_{w1}}{\gamma_{w2}}d\theta$$

When $PM=L$, calculation formula is as follows,

$$\overline{I_1M} = \overline{PI_1} - \overline{PM} = \gamma_{w1} \bullet \sin\alpha_w - L$$

$$\overline{I_2M} = \overline{PI_2} - \overline{PM} = \gamma_{w2} \bullet \sin\alpha_w + L$$

$$L = \sqrt{\gamma_{a2}{}^2 - \gamma_{b2}{}^2} - \gamma_{w2} \bullet \sin\alpha_w$$

Refer to Fig. 18. Specific sliding for each part of Tooth profile.

Table 18. Specific sliding for Involute gear

	Specific sliding of Addendum flank	Specific sliding of Dedendum flank
Gear 1	$\delta_{a1} = \dfrac{1+\dfrac{\gamma_{w1}}{\gamma_{w2}}}{\dfrac{\gamma_{w1}}{L}\sin\alpha_w + 1}$	$\delta_{f1} = \dfrac{1+\dfrac{\gamma_{w1}}{\gamma_{w2}}}{\dfrac{\gamma_{w1}}{L}\sin\alpha_w - 1}$
Gear 2	$\delta_{a2} = \dfrac{1+\dfrac{\gamma_{w1}}{\gamma_{w2}}}{\dfrac{\gamma_{w1}}{L}\sin\alpha_w + \dfrac{\gamma_{w1}}{\gamma_{w2}}}$	$\delta_{f2} = \dfrac{1+\dfrac{\gamma_{w1}}{\gamma_{w2}}}{\dfrac{\gamma_{w1}}{L}\sin\alpha_w - \dfrac{\gamma_{w1}}{\gamma_{w2}}}$

As for Involute gear, refer to Fig. 31 for sliding contact to all areas except area of intermeshing pitch point. The Specific sliding increases as teeth moves away from Pitch point
When Contact ratio increases for Involute tooth profile, condition of Specific sliding will have a tendency to decrease.

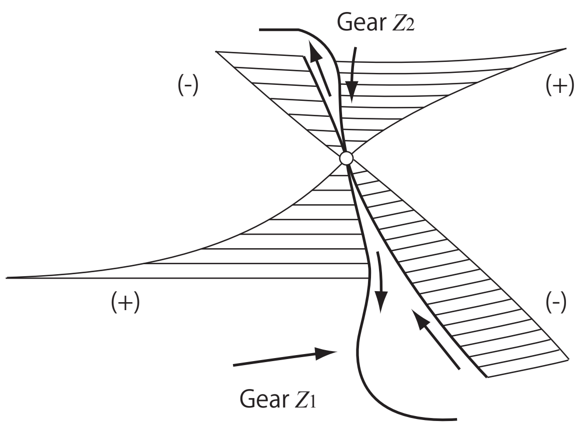

Fig. 31 Distribution of Specific sliding