1.8 Rack shift of the gear

Undercut

When Number of teeth is below minimum as shown in Fig. 23, part of dedendum is no longer an Involute curve but will look like a shape scooped out by cutter tool.

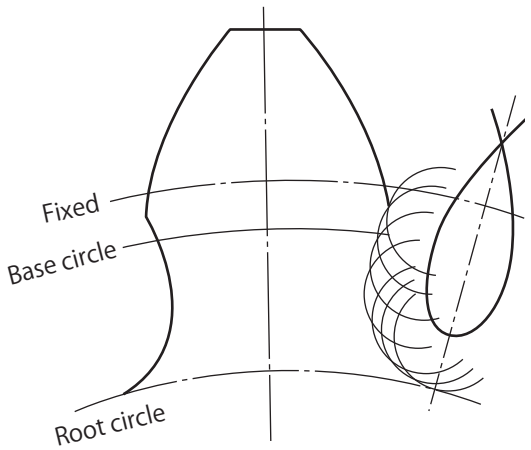
Refer to drawing, when Involute curve shows the scooped out shape condition from Base circle (Tooth tip side), it is called **Undercut**.

Gear with undercut has low strength of Dedendum and provides bad influence to gear contact due to shortened Involute curve.

Calculation formula for minimum number of teeth (z) to prevent undercut is as follows,

$$z = \frac{2}{\sin^2 \alpha_0} \qquad (\alpha_0: \text{Cutter pressure angle})$$

Condition of Undercut generally appears when Number of teeth is 17 or less and pressure angle of gear is 20°. According to DIN standard, minimum Number of teeth is 14 accepting slight Undercut which may cause no serious influence.



(The Trochoid curve line on the right hand side is the centre locus of roundness of cutter of rack tool. (radius of roundness γ_f=0.375m=7.5)

Fig. 23 Undercut

Profile Shifted Gear

(1) The Summary of Profile Shifted Gear

Using a rack tool (for example, hob cutter) to fabricate Profile Shifted Gear is to achieve the following purposes.

1) Prevent condition of Undercut for gear with less than minimum Number of teeth.

2) When there is deviation or failure for centre distance, fabricate a modified gear to correct the fault centre distance.

(3) Adjust distribution of Tooth thickness for gear pair to achieve equal gear strength.

4) Adjust to suitable contact ratio to lessen gear noise level and/or trapping of pump gear.

5) Take into consideration the wear of flank to adjust Specific sliding. (Another theory states that Specific sliding and wear are not proportional.)

It is possible to adjusting gear by item 2) to control helix angle of Helical gear. However it is necessary to provide thrust bearing in axis direction to countermeasure force (thrust force) occurring in Helical gear.

When design multi engagement between axes with different gear ratio, items 2) is also useful (for example, speed reducer).

Generally, Positive profile shift (+) is the method of gear fabrication where Reference pitch line of Rack type cutter shifts x-times of module toward outer radius direction from Reference pitch. The Negative profile shift (-) is that Reference pitch line of Rack type cutter shifts x-times of module towards inner radius direction from Reference pitch. $x.m$ is commonly called the **Amount of rack shift** where x is called **Rack shift coefficient**. (Please refer to Fig. 24).

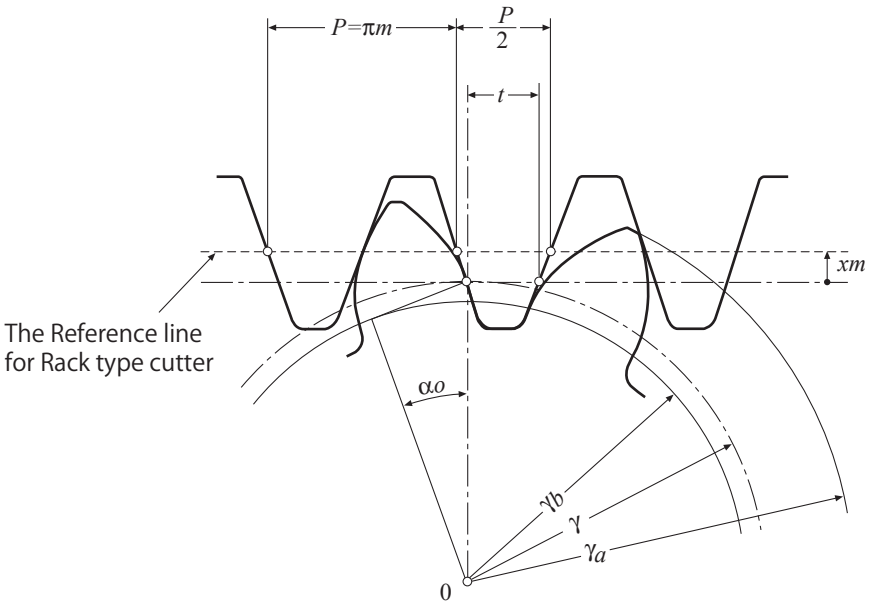
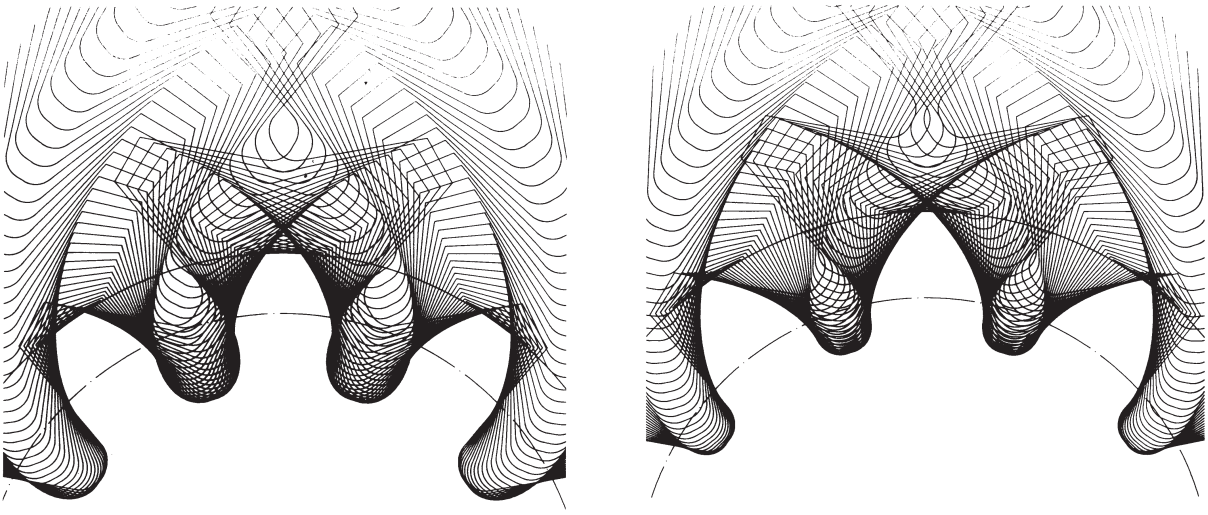


Fig. 24 Rack shift for Spur gear



Spur gear with full depth tooth
(Rack shift coefficient x=0)

Positive (+) profile shifted gear
(Rack shift coefficient x=0.5)

Fig. 25 Profile shifted gear (Examples of Positive and Negative profile shifted gear, Number of teeth is 12z)

Note: (1) Adopted the old Standard term.

Limitation of Pointed tooth tip

When increase the positive amount of Rack shift, area of top land is gets narrower and soon, Tooth profile becomes sharp.

A sharp pointed Tooth profile has insufficient tooth depth, thus Tooth tip of Mating gear may interfere with Root of tooth causing proper assembly and smooth gear rotation to be impossible. Therefore Rack shift of Top land exceeding zero is not advisable.

To calculate Top land 's' of Spur gear by the following formula,

$$s = m(z + 2 + 2x) \bullet \left\{ \left(\frac{\pi}{2} + 2x\tan\alpha_0 \right) \bullet \frac{1}{z} - (\mathrm{inv}\,\alpha_a - \mathrm{inv}\,\alpha_0) \right\}$$

For easy reference, please refer to Table 14 for area of formed gear with Pressure angle 20°.

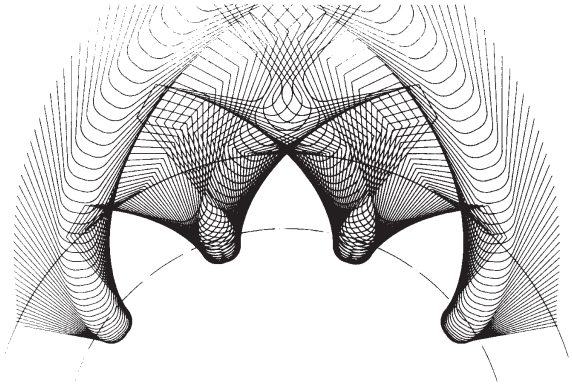


Fig. 26 Pointed tooth tip

Calculation for Rack shift coefficient.

(1) Rack shift coefficient to prevent Undercut.

Undercut is sure to occur when Number of teeth is 17 or below with Pressure angle 20°. Prevent Undercut using theoretical Rack shift coefficient by following calculation formula.

$$x = \frac{17 - z}{17} \qquad (z\text{: Practical number of teeth})$$

Practical number of teeth 14 z is available to use for DIN standard, calculation formula of DIN is defined as follows.

$$x = \frac{14 - z}{17} \qquad (z\text{: Practical number of teeth})$$

Theoretical Rack shift coefficient for Spur gear with Number of teeth 10z with Pressure angle 20° is by following formula

$$x = \frac{17 - 10}{17} = 0.412$$

(Please check for occurrence of sharp pointed tooth top tip using Table 14.)

Practical rack shift coefficient is obtained by following calculation.

$$x = \frac{14 - 10}{17} = 0.235$$

(2) Rack shift coefficient to adjust Centre distance

Below is the explanation using examples.

For example, calculate Rack shift coefficient for adjustable gear with Centre distance of 80.5mm (Proper distance is 80.0mm) with: Gear: Spur gear,

Pressure angle:20°,

Module: 2.0mm,

Number of teeth for Pinion: 20z,

Number of teeth for Gear: 60z,

Centre distance modification coefficient

$$y = (a' - a)/m$$
$$= (80.5 - 80)/2$$
$$= 0.25$$

$$y = \frac{z_1 + z_2}{2}\left(\frac{\cos\alpha_0}{\cos\alpha_w} - 1 \right) \quad \text{therefore}$$

$$\cos\alpha_w = \frac{\cos\alpha_0}{\frac{2 \bullet y}{z_1 + z_2} + 1} = \frac{\cos 20°}{\frac{2 \bullet 0.25}{20 + 60} + 1}$$

$$= 0.933856$$
$$\alpha_w = 20.955894°$$
$$\mathrm{inv}\,\alpha_w = \tan\alpha_w - \alpha_w$$
$$= \tan 20.955894° - 20.955894° \bullet \pi/180$$
$$= 0.0172317$$

$$\mathrm{inv}\,\alpha_w = 2 \bullet \tan\alpha_0 \bullet \left(\frac{x_1 + x_2}{z_1 + z_2} \right) + \mathrm{inv}\,\alpha_0 \quad \text{therefore}$$

Sum of Rack shift coefficient

$$x_1 + x_2 = \left(\frac{\mathrm{inv}\,\alpha_w - \mathrm{inv}\,\alpha_0}{2 \bullet \tan\alpha_0} \right) \bullet (z_1 + z_2)$$

$$= \frac{0.0172317 - 0.0149044}{2 \bullet \tan 20°} = 0.2557$$

a' : Actual centre distance (mm)
a : Proper centre distance (mm)
z_1 : Number of teeth for Pinion
z_2 : Number of teeth for Gear
α_0 : Pressure angle of Cutter (°)
α_w :pressure angle (°)
y : Centre distance increment coefficient
x_1 : Rack shift coefficient for Pinion
x_2 : Rack shift coefficient for Gear
$\mathrm{inv}\alpha_0$: Functional involute for Cutter pressure angle
$\mathrm{inv}\alpha_0 = \tan\alpha_0 - \alpha_0$
$\mathrm{inv}20° = 0.0149044$
(The lastα_0 is in Radian Unit)

You may provide the sum (0.2557) of this Rack shift coefficient to Pinion only or can divide between Gear and Pinion.

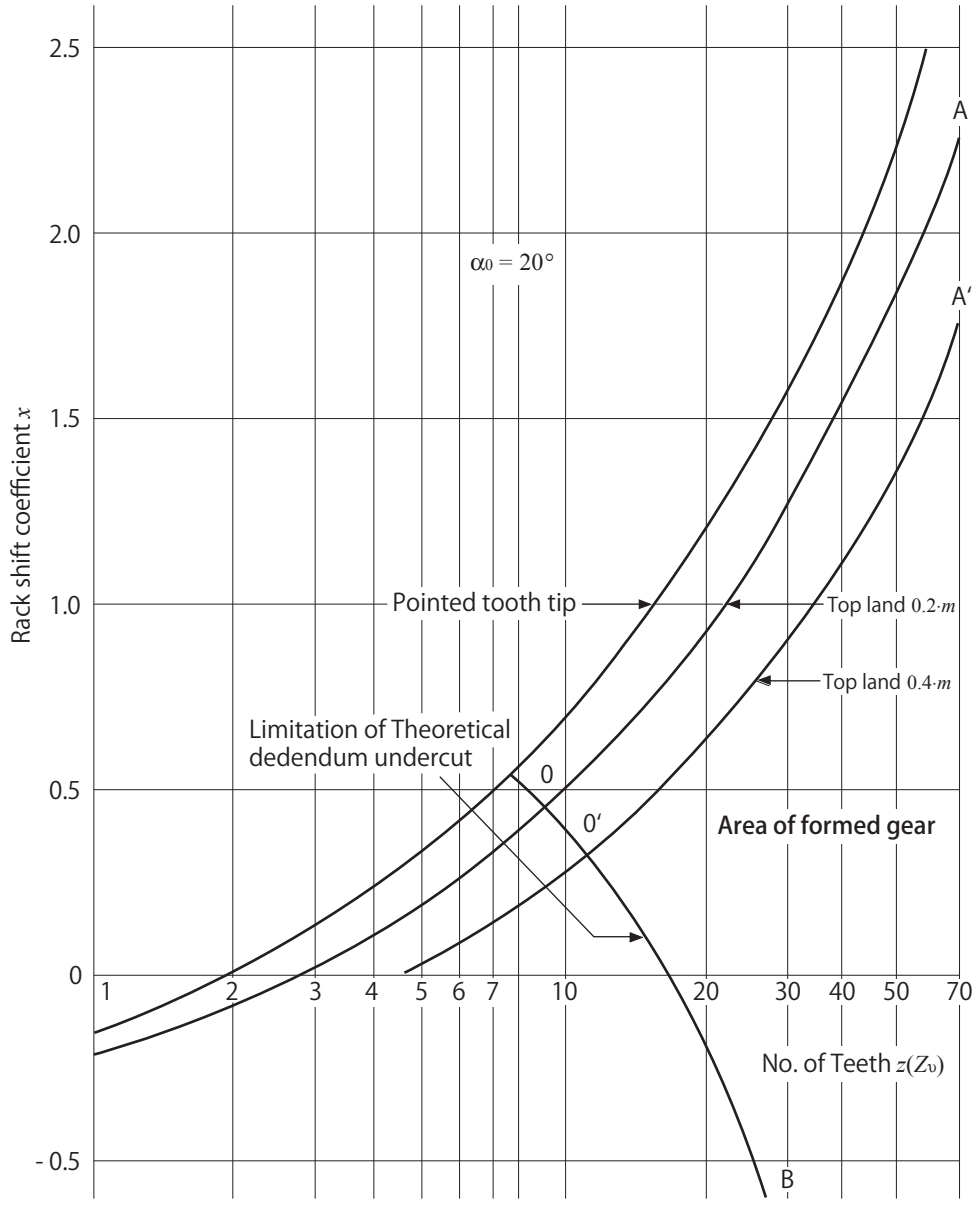
(3) Guidelines for determining Rack shift coefficient.

Rack shift to positive side is mainly designed for Pinion. It is necessary to check that the calculated Rack shift coefficient does not cause pointed tooth tip.

If design causes pointed tooth tip, reduce amount of Rack shift coefficient to Pinion and offset amount to Gear.

As for Rack shift to negative side, it is necessary to check for Undercut. If Undercut should occur, offset the Negative rack shift coefficient to mating gear.

Refer to Table 14 to shown the area of formed gear with pressure angle 20°.



Curved line A : Top land changed to 0.2 • m by Rack shift coefficient and Number of teeth.

Curved line A' : Top land changed to 0.4 • m by Rack shift coefficient and Number of teeth.

Curved line B : Rack shift coefficient and Number of teeth for Limitation of Theoretical dedendum undercut.

* For Helical gear, use horizontal axis in chart for Virtual number of teeth of spur gear Zv.

$$Zv = Z / \cos^3\beta$$

Table 14. Area of formed gear (pressure angle 20°)

Note (1) Adopted old gear terms.

The features of Tooth profile 05

Tooth profile of KG STOCK GEARS (Number of teeth from 8z to 11z) has been adopted by type 05 in DIN standard.

Tooth profile type 05 has its Rack shift coefficient fixed to plus (+) 0.5. Adjust Addendum by shortening coefficient x module ($\kappa.m$) to fabricate smaller Outside diameter, as the Bottom clearance have a tendency to be narrow.

The calculation of Rack shift for Number of teeth ranging from 8z to 11z for KG STOCK GEARS is as follows,

Calculation formula for Working pressure angle α_w is as follows:

$$\text{inv}\,\alpha_w = 2\tan\alpha\left(\frac{x_1+x_2}{z_1+z_2}\right)+\text{inv}\,\alpha$$

Hereby

z_1=No. of teeth for Pinion

z_2= No. of teeth for Gear

z_1=Rack shift coefficient for Pinion

x_2=Rack shift coefficient for Gear

α_0= Pressure angle (Cutter pressure angle)

inv= Involute function

$\text{inv}\alpha=\tan\alpha-\alpha$

(Refer to page 164-167 for the Involute function table)

Centre distance modification coefficient y is as follows:

$$y=\frac{z_1+z_2}{2}\left(\frac{\cos\alpha}{\cos\alpha_w}-1\right)$$

Centre distance a_x is following formula:

$$a_x=\left(\frac{z_1+z_2}{2}+y\right)m$$

Hereby

m=module

Working pitch diameter d'_1 and d'_2 is by following formula:

$$d'_1=2\,a_x\left(\frac{z_1}{z_1+z_2}\right)$$

$$d'_2=2\,a_x\left(\frac{z_2}{z_1+z_2}\right)$$

Reference diameter d_1 and d_2 is by following formula:

$$d_1=z_1m$$

$$d_1=z_2m$$

Tip (Outside) diameter d_{ax} is following formula:

$$d_{ax}=2m\left(\frac{z+3}{2}-\kappa\right)$$

Hereby

κ=Truncation coefficient

$$\kappa m=\left[x_1+x_2-\frac{z_1+z_2}{2}\left(\frac{\cos\alpha}{\cos\alpha_w}-1\right)\right]m$$

When Addendum of cutter is module 1.25, Bottom clearance (minimum amount) is module 0.21.

The Centre distance for number of teeth 8z and 8z is as follows,

(Rack shift coefficient x=0.5)

$$a_x/m=8.7788\text{mm}$$

The centre distance for number of teeth 10z and 10z is as follows.

(Rack shift coefficient x=0.5)

$$a_x/m=10.8043\text{mm}$$

The above calculations are for module 1.0. Example for module is 2.0 with number of teeth 8z and 8z are engaged, centre distance ax based on above a_x/m = 8.778mm is as follows:

$$a_x=8.7788\times2$$
$$=17.5576\text{mm}$$

Mating gear with other Number of teeth of KG STOCK GEARS is available.

With regards to the tooth profile of type 05 for the Rack Shift Coefficient quoted by Gear Industry Volume No.54, “German Gear Standard” (DIN 3994 and 3995)

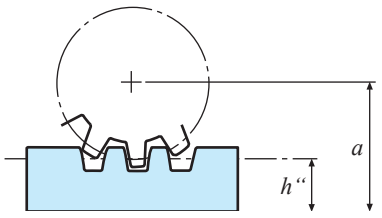
The Centre distance between KG Rack shifted spur gear and KG STOCK GEARS

Usage of below comparison table: Where module is 1.0, calculate the centre distance a_x multiply by module.

No. of teeth

No. of teeth / No. of teeth	8	9	10	11
8	8.779	9.286	9.792	10.298
9	9.286	9.792	10.299	10.804
10	9.792	10.299	10.804	11.310
11	10.299	10.804	11.310	11.815
12	10.437	10.939	11.441	11.943
13	10.939	11.441	11.943	12.445
14	11.441	11.953	12.445	12.946
15	11.943	12.445	12.946	13.448
16	12.445	12.946	13.448	13.949
17	12.946	13.448	13.949	14.451
18	13.448	13.949	14.451	14.952
19	13.949	14.451	14.952	15.453
20	14.451	14.952	15.453	15.954
21	14.952	15.453	15.954	16.455
22	15.453	15.954	16.455	16.956
23	15.954	16.455	16.956	17.457
24	16.455	16.956	17.457	17.958
25	16.956	17.457	17.958	18.459
26	17.457	17.958	18.459	18.960
27	17.958	18.459	18.960	19.461
28	18.459	18.960	19.461	19.962
29	18.960	19.461	19.962	20.463
30	19.461	19.962	20.463	20.963
32	20.463	20.963	21.464	21.965
34	21.464	21.965	22.465	22.966
35	21.965	22.465	22.966	23.467
36	22.465	22.966	23.467	23.967
38	23.467	23.967	24.468	24.968
40	24.468	24.968	25.469	25.969
42	25.469	25.969	26.470	26.970
44	26.470	26.970	27.471	27.971

No. of teeth / No. of teeth	8	9	10	11
45	26.970	27.471	27.971	28.472
46	27.471	27.971	28.472	28.972
48	28.472	28.972	29.473	29.973
50	29.473	29.973	30.473	30.974
52	30.473	30.974	31.474	31.974
54	31.474	31.974	32.475	32.975
55	31.974	32.475	32.975	33.475
56	32.475	32.975	33.475	33.976
58	33.475	33.976	34.476	34.976
60	34.476	34.976	35.477	35.977
62	35.477	35.977	36.477	36.977
64	36.477	36.977	37.478	37.978
65	36.977	37.478	37.978	38.478
66	37.478	37.978	38.478	38.979
68	38.478	38.979	39.479	39.979
70	39.479	39.979	40.879	40.979
72	40.479	40.979	41.480	41.980
75	41.980	42.480	42.980	43.480
80	44.481	44.981	45.481	45.981
84	49.482	46.982	47.482	47.982
85	46.982	47.482	47.982	48.482
90	49.483	49.983	50.483	50.983
95	51.983	52.483	52.984	53.484
96	52.483	52.984	53.484	53.984
100	54.484	54.984	55.484	55.985
105	56.985	57.485	57.985	58.485
108	58.485	58.985	59.485	59.985
110	59.485	59.985	60.485	60.986
112	60.485	60.986	61.486	61.986
115	61.986	62.486	62.986	63.486
120	64.486	64.987	65.487	65.987



Centre distance between KG-Rack shifted spur gear and KG-Rack

$$a = h'' + \frac{m \times z}{2} + xm$$

Hereby

- a : Centre Distance (Distance from Datum of Rack to Centre of KG-Spur gear)
- h'' : Datum line of Rack (Refer to page 259)
- m : Module
- x : Rack shift coefficient
- z : Number of teeth

(Module 1.0 and above
For Number of teeth 8 to 11, x =0.5
For Number of teeth 12 and above, x=0)